REPORT DOCUMENTATION PAGE

Form Approved OMB NO. 0704-0188

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
03-11-2010	Final Report	15-Sep-2009 - 14-Jun-2010

4. TITLE AND SUBTITLE
The Role of Channel Distribution Information for Interference Management and Network Performance Enhancement

5a. CONTRACT NUMBER: W911NF-09-1-0494

5b. GRANT NUMBER:

5c. PROGRAM ELEMENT NUMBER: 611102

5d. PROJECT NUMBER:

5e. TASK NUMBER:

5f. WORK UNIT NUMBER:

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8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSOR/MONITOR'S ACRONYM(S): ARO

11. SPONSOR/MONITOR'S REPORT NUMBER(S): 57328-NS-II.2

12. DISTRIBUTION AVAILABILITY STATEMENT
Approved for Public Release; Distribution Unlimited

13. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not contrued as an official Department of the Army position, policy or decision, unless so designated by other documentation.

14. ABSTRACT
One of the key technologies proposed for next generation commercial and military communications systems is the use of multiple antennas at both the transmitting and receiving nodes of the network. The additional antenna elements provide degrees of freedom that can provide spatial filtering, spatial multiplexing and diversity gain. These advances can be applied to increase link gain, provide interference management, mitigate hostile jammers and facilitate multi-packet reception.

15. SUBJECT TERMS
channel statistics, interference management, covariance feedback, beamformers,multiuser MIMO, outage probability, network scheduling

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	15. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
UU	UU	UU	UU		James Zeidler
					19b. TELEPHONE NUMBER: 858-534-5369

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std. Z39.18

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
2010	**N/A**	**-**

4. TITLE AND SUBTITLE **Outage-Efficient Strategies for Multiuser MIMO Networks with Channel Distribution Information**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **University of California - San Diego 9500 Gilman Drive MC 0934 La Jolla, CA 92093 -0934**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S) 11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
The original document contains color images.

14. ABSTRACT
In this work, we examine single user and multiuser Multiple-Input Multiple-Output (MIMO) beamforming networks with Channel Distribution Information (CDI). Since CDI changes infrequently compared to Channel State Information (CSI), algorithms based on CDI can achieve significant savings in feedback compared to algorithms based on CSI. With CDI, we can only guarantee quality of service for a specified outage probability in the network. Assuming correlated Rayleigh fading on all the links, we derive a closed-form expression for the outage probability. Then, using this expression, we derive algorithms for joint transmit/receive beamforming and power control to minimize the weighted sum power in the network while guaranteeing these outage probabilities. For both singleuser and multiuser MIMO scenarios, we present optimal algorithms under the Kronecker model assumption, and we present near-optimal algorithms assuming general correlation structures on the links. We then show that using these algorithms based on CDI, if we are willing to accept given outages on the links, we can achieve comparable power usage in the network relative to algorithms based on CSI.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**SAR**	**21**	

Report Title

The Role of Channel Distribution Information for Interference Management and Network Performance Enhancement

ABSTRACT

One of the key technologies proposed for next generation commercial and military communications systems is the use of multiple antennas at both the transmitting and receiving nodes of the network. The additional antenna elements provide degrees of freedom that can provide spatial filtering, spatial multiplexing and diversity gain. These advances can be applied to increase link gain, provide interference management, mitigate hostile jammers and facilitate multi-packet reception.

In this project we defined how the the channel statistics can be utilized in place of the instantaneous channel state informaton to limit the amount of power that must be transmitted between nodes to improve overall network performance by maintaining stable beamforming vectors over the time scales defined by channel distribution information rather than the reduced time scale associated with the channel coherence time.

With CDI, one can only guarantee quality of service for a specified outage probability. This creates a tradeoff that can be beneficial to the overall network. Closed form expressions for the outage probabilities were derived and given those expressions, algorithms were derived that minimize the weighted sum power in the network for a specified outage probability.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Sagnik Ghosh, Bhasker D. Rao, and James .R. Zeidler, “Outage Efficient Strategies for Multiuser MIMO Networks with Channel Distribution Information”, IEEE Transactions on Signal Processing, December 2010 in press.

Number of Papers published in peer-reviewed journals: 1.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 0.00

(c) Presentations

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

Peer-Reviewed Conference Proceeding publications (other than abstracts):

SS Tan, A. Anderson and J.R. Zeidler, “The Role of Channel Distribution Information in the Cross-layer Design of an Opportunistic Scheduler for MIMO Networks” IEEE Asilomar Conference, November 2010 (finalist for best student paper award)

S. Ghosh, B.R. Rao, and J. R. Zeidler, ``Outage-optimal transmission in multiuser-mimo kronecker channels," in Proc. 2010 IEEE Intl. Conf. Acoustics, Speech, and Signal Processing, San Diego, March 2010.

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 2

(d) Manuscripts

Number of Manuscripts: 0.00

Patents Submitted

Patents Awarded

Awards

Finalist Best Student Paper Award, IEEE Asilomar Conference, Pacific Grove, CA November 2010

Graduate Students

NAME	PERCENT_SUPPORTED
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FTE Equivalent:	**0.40**
Total Number:	**1**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
James Zeidler	0.20	No
FTE Equivalent:	**0.20**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics
This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:	0.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:......	0.00
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Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):......	0.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:......	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:......	0.00

Names of Personnel receiving masters degrees

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Total Number:

Names of personnel receiving PHDs

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Names of other research staff

NAME	PERCENT_SUPPORTED
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Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Outage-Efficient Strategies for Multiuser MIMO Networks with Channel Distribution Information

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***Abstract*— In this work, we examine single user and multiuser Multiple-Input Multiple-Output (MIMO) beamforming networks with Channel Distribution Information (CDI). Since CDI changes infrequently compared to Channel State Information (CSI), algorithms based on CDI can achieve significant savings in feedback compared to algorithms based on CSI. With CDI, we can only guarantee quality of service for a specified outage probability in the network. Assuming correlated Rayleigh fading on all the links, we derive a closed-form expression for the outage probability. Then, using this expression, we derive algorithms for joint transmit/receive beamforming and power control to minimize the weighted sum power in the network while guaranteeing these outage probabilities. For both single-user and multiuser MIMO scenarios, we present optimal algorithms under the Kronecker model assumption, and we present near-optimal algorithms assuming general correlation structures on the links. We then show that using these algorithms based on CDI, if we are willing to accept given outages on the links, we can achieve comparable power usage in the network relative to algorithms based on CSI.**

***Index Terms*— covariance feedback, beamformers, multiuser MIMO, outage probability, Channel State Information, Channel Distribution Information; EDICS: MSP-CAPC, MSP-MULT, WIN-ADHC, WIN-PHYL**

I. Introduction

Multiuser multiple-input multiple-output (MU-MIMO) networks have generated much research interest in recent years. As the community has come to have a greater understanding of MIMO point-to-point links, recent work has involved looking at how multiple antennas can be utilized to reduce interference in multiuser scenarios. Some useful surveys and texts on the topic are [1]-[4]. To achieve maximum throughput and reliability in the MU-MIMO network, all the nodes need to have perfect Channel State Information (CSI) of all the links in the network. With this knowledge, utilizing intelligent beamforming, interference in the system can be minimized and optimal rates can be achieved.

This assumption may be feasible in small networks where the channels change slowly. The nodes then have time to estimate their channel and feed back their CSI to the rest of the nodes before transmitting their data. However, in reality, many networks are large, mobile, and operate in dynamic environments. Thus, the channels change too fast for the CSI to be fed back to all the nodes–by the time all the CSI has reached every node, the information is already outdated. Thus, much work has focused on finding transmission schemes that do not rely on full CSI. The simplest transmission scheme assumes no knowledge about the channel structure and employs equal power allocation on all the antennas [1]. This approach has the advantage of having no feedback overhead, but yields lower throughput and gives no insight into the transmit power required for reliable multiuser communication as compared to schemes that utilize feedback. Another transmission scheme uses limited CSI, where nodes feed back their CSI only to nearby nodes [7]. This approach assumes that the nearby nodes dominate the interference, and should therefore yield near-optimal performance. However, even in point-to-point systems, very low channel coherence times make full CSI feedback infeasible. Many schemes thus utilize quantized forms of CSI to reduce feedback [5][6]. Still, these approaches suffer from having to feed back information every time the channel changes.

Another approach, then, is to utilize the *channel statistics*, or Channel Distribution Information (CDI), to enhance communication. Since it takes into account the randomness in the channel, CDI is more robust to

This work was sponsored by the U. S. Army Research Office under the Multi-University Research Initiative (MURI) Grant # W911NF-04-1-0224 and the USARAC Grant # W911NF-09-1-0494.

small channel coherence times and is thus valid for much longer than CSI. In addition, given the trend towards more location-based services, statistical data based on node location can also be collected and stored *a priori*. Such location-based data eliminates the need for real-time channel feedback. CDI has been combined with various CSI schemes to improve performance and reduce feedback in [8]-[10]. In [8], CDI is used to aid in SINR feedback, and in [10], CDI is used in combination with channel norm feedback to exploit multiuser diversity. In [9], CDI is utilized for more efficient beamformer codebook design. While these schemes use CDI to reduce CSI feedback, channel information must still be fed back every time the channel changes. Thus, much previous work has focused on utilizing CDI without the aid of CSI.

Previous work using only CDI focuses on maximizing ergodic capacity. In [11], optimal transmission strategies are given for single-user multiple-input single-output (SU-MISO) channels when either the mean channel information or covariance channel information is known. The capacity-optimal input covariance for the single-user MIMO (SU-MIMO) channel with mean and covariance channel information is presented in [12]. This result is extended to the MIMO multiple access channel in [13]. Suboptimal solutions for the MIMO broadcast channel are given in [14]. These works focus on average capacity, but this metric relies on averaging over good and bad channels to achieve the desired throughput. Thus, packet delay is ignored in such schemes, and average capacity analysis is not applicable to delay-sensitive applications.

This work seeks to account for this delay problem by looking at the *outage* of the links in the network. Due to the randomness in the channel, reliable transmission cannot be achieved all the time using just CDI. Consequently, in this work we guarantee a certain signal-to-interference-plus-noise ratio (SINR) with a specified probability on all the links. Specifically, this work looks to jointly optimize the power allocation and transmit/receive beamformers to minimize the power used in the network while meeting these outage SINR constraints. While many works in the literature assume the channels experience independent identically-distributed (i.i.d.) Rayleigh fading across the transmit and receive antennas for analytic convenience, much work has been done showing the insufficiency of this model [15]-[18]. Therefore, this work assumes all links in the network experience *correlated* Rayleigh fading. Under this assumption, the expression for the outage probability on each link is derived. This expression is then applied to the SU-MIMO channel, and an algorithm for joint transmit and receive beamforming and power control is presented. We then study the MU-MIMO system, where algorithms for joint transmit and receive beamforming and power control are discussed. Under the Kronecker model assumption [15], an optimal algorithm is given, and for general correlation structures a close-to-optimal algorithm is derived. Then, using the derived CDI algorithms, we show that while we accept some loss on the links in the network, we can achieve power consumption levels attained by CSI schemes.

The paper is organized as follows: in Section II, the problem formulation is given and discussed, and the expression for outage probability is derived. Section III discusses the SU-MIMO problem. Section III-A finds the closed-form expression for the optimal power control for a fixed set of beamformers. Section III-B derives the optimal beamformers for the Kronecker model, and III-C derives an algorithm for beamforming under general Rayleigh fading. Section IV discusses the MU-MIMO problem. First, IV-A derives an optimal power control algorithm with a fixed set of beamformers. Section IV-B then derives the optimal beamformers for the Kronecker model, and Section IV-C discusses a good suboptimal solution for the beamformers assuming general Rayleigh fading. We compare our derived CDI algorithms to CSI algorithms in Section V, and we summarize our results in Section VI.

In this work, the following notation is used: italicized letters indicate scalars (e.g. p_l), lower-case bold letters indicate vectors (e.g. $\mathbf{v}$), and upper-case bold letters indicate matrices (e.g. $\mathbf{A}$). Furthermore, $(\bullet)^H$ indicates the Hermitian operator, and $(\bullet)^*$ indicates the conjugate operator.

II. Problem Formulation

A. System Model

This work considers time-varying MIMO channels for many users in a network. Consider a MIMO network with L transmit-receive pairs. At link l, the transmitter sends the symbol $s_l(t)$ to the receiver. The transmitter uses unit-norm beamforming vector $\mathbf{v}_l(t)$ to precode the signal, and transmits with power $p_l(t)$. The receiver employs the linear unit-norm beamformer $\mathbf{u}_l(t)$ to combine the signal. The channel from transmitter i to receiver l is given by $\mathbf{H}_{li}(t)$. The noise $N_l(t)$ is distributed as a complex circular Gaussian, and represents the combined noise after applying the receive beamforming vector to the incoming signal. The l^{th} received signal is thus given by

$$\begin{aligned} r_l(t) &= \sqrt{p_l(t)}[\mathbf{u}_l^H(t)\mathbf{H}_{ll}(t)\mathbf{v}_l(t)]s_l(t) + N_l(t) \\ &+ \sum_{i\neq l}^{L} \sqrt{p_i(t)}[\mathbf{u}_l^H(t)\mathbf{H}_{li}(t)\mathbf{v}_i(t)]s_i(t) \end{aligned}$$

In schemes that use perfect CSI, a block-fading model is assumed, so the channel stays constant over each block. Then, for notational convenience, the time variable will be dropped for the channel, power allocations, and beamformers. To further simplify notation, define $G_{li} = |\mathbf{u}_l^H \mathbf{H}_{li} \mathbf{v}_i|^2$ as the beamforming channel gain from the transmitter on link i to the receiver at link l and $\sigma_{N_l}^2$ as the noise power for the l^{th} link. Then, under this model, the SINR Γ_l on each link can be shown to be

$$\Gamma_l = \frac{p_l G_{ll}}{\sum_{i \neq l} p_i G_{li} + \sigma_{N_l}^2} \tag{1}$$

If perfect CSI is available, to ensure a reliable link is available to all nodes in the network, each link has an SINR constraint: Γ_l must be greater than a threshold γ_l. The goal is then to minimize the power consumed by the network while meeting all the SINR constraints. The cost function considered in this work is the *weighted sum power*. In this setup, each link l in the network incurs some cost $w_l > 0$ to transmit across its link. An example of a network with varying power costs on the links are networks with varying battery life at the transmitters. For minimizing non-weighted sum power, $w_l = 1$ for $l = 1, \ldots, L$.

To compact notation, define the weighting and power vectors as $\mathbf{w} = \{w_1, \ldots, w_L\}$ and $\mathbf{p} = \{p_1, \ldots, p_L\}$, respectively. The beamforming matrices are defined as $\mathbf{U} = \{\mathbf{u}_1, \ldots, \mathbf{u}_L\}$ and $\mathbf{V} = \{\mathbf{v}_1, \ldots, \mathbf{v}_L\}$. The optimization problem for having perfect CSI, formulated and discussed in [7], can then be stated as follows:

$$\begin{aligned} &\min_{\mathbf{p} \geq 0, \mathbf{U}, \mathbf{V}} \mathbf{w}^T \mathbf{p} \\ &\text{s.t.} \quad \Gamma_l \geq \gamma_l, l = 1, \ldots, L \end{aligned} \tag{2}$$

For a fixed set of channel matrices $\mathbf{H}_{li}$'s, this problem can be solved and will give a set of power allocations and beamformers for the transmitters and receivers in the network. Then, for every change in the $\mathbf{H}_{li}$'s, all the transmit and receive beamformers must be updated, and the power allocation scheme changes. In many networks, the feedback required for these changes in the channel is unrealistic due to rapidly-varying CSI. Thus, this work considers a network where instantenous CSI is unavailable, but CDI is available. When only CDI is known, the exact $\mathbf{H}_{li}$'s are not known–instead, they are assumed to be a random variable drawn from a complex-normal distribution:

$$\text{vec}(\mathbf{H}_{li}) \sim CN(0, \mathbf{\Sigma}_{li})$$

The channel covariance matrices, given by the $\mathbf{\Sigma}_{li}'s$, comprise the CDI of the network. This work will consider the case where the channel varies, but the statistics stay constant. Under this model, the expression for SINR given in (1) becomes a random variable since it depends on the channel. The constraints in (2) can then no longer be written as the SINR on link l always exceeding some threshold γ_l–since the SINR is now random, it will drop below γ_l with some probability. Therefore, these absolute constraints change to outage constraints, and links are allowed to have an SINR below their thresholds for specified probabilities. Mathematically, the constraint on link l in (2) becomes $\Pr(\Gamma_l \leq \gamma_l) \leq \alpha_l$, where α_l is the probability that the link is in outage. The main optimization problem using CDI can then be formulated:

$$\begin{aligned} &\min_{\mathbf{p} \geq 0, \mathbf{U}, \mathbf{V}} \mathbf{w}^T \mathbf{p} \\ &\text{s.t.} \quad \Pr(\Gamma_l \leq \gamma_l) \leq \alpha_l, l = 1, \ldots, L \end{aligned} \tag{3}$$

In this formulation, the constraints ensure that the l^{th} link will be active with probability $(1 - \alpha_l)$. In order to solve this optimization problem, the first step is to derive a closed-form expression for the outage probability, $\Pr(\Gamma_l \leq \gamma_l)$.

B. Derivation of Outage Probability

Two steps are used in deriving this closed-form expression of the outage probability. The first is to express Γ_l as a ratio of an exponential random variable to a weighted sum of exponential random variables plus some constant. The second step is to express $\Pr(\Gamma_l \leq \gamma_l)$ in terms of the moment-generating function for a sum of weighted exponential random variables to obtain the final result.

Without loss of generality, the expression for outage probability of the first user will be shown (treating all other users as interference). Furthermore, for convenience, the user subscript l will be dropped (G_{11} becomes G_1, $\sigma_{N_l}^2$ becomes σ_N^2 and so on). Thus, the expression for SINR being considered is

$$\Gamma = \frac{p_1 G_1}{\sum_{i \neq 1} p_i G_i + \sigma_N^2} \tag{4}$$

The following Lemma will also be used in the proof of the theorem:

Lemma 1: If $\mathbf{u}$ and $\mathbf{v}$ are unit-norm vectors and $\mathbf{H}$ is complex circular Gaussian matrix distributed as $\text{vec}(\mathbf{H}) \sim CN(0, \mathbf{\Sigma})$, then

$$\frac{|\mathbf{u}^H \mathbf{H} \mathbf{v}|^2}{(\mathbf{v}^* \otimes \mathbf{u})^H \mathbf{\Sigma} (\mathbf{v}^* \otimes \mathbf{u})} \sim \chi_2^2 \tag{5}$$

The proof is given in Appendix I. The theorem can then be stated as follows:

Theorem 1: In a MU-MIMO network where all links experience correlated Rayleigh fading and the transmitter and receiver both employ linear beamforming, the expression for outage probability for the SINR for user 1 is given by

$$\rho_{out} = \Pr(\Gamma \leq \gamma) = 1 - e^{-\frac{\gamma}{2}\frac{\sigma_N^2}{c_1 p_1}} \prod_{i=2}^{L} \left(1 + \gamma \frac{c_i p_i}{c_1 p_1}\right)^{-1}, \quad (6)$$

where

$$c_i = (\mathbf{v}_i^* \otimes \mathbf{u}_1)^H \mathbf{\Sigma}_i (\mathbf{v}_i^* \otimes \mathbf{u}_1)$$

Proof: Step 1: Get ρ_{out} into the following form:

$$\rho_{out} = \Pr(\Gamma \leq \gamma) = \Pr(\frac{X}{Y + \sigma^2} \leq \gamma), \quad (7)$$

where X is an exponential random variable and Y is a weighted sum of independent exponential random variables. Applying *Lemma 1* to the G_l terms in the expression for Γ in Eqn. (4) gives

$$\Gamma = \frac{c_1 p_1 Z_1}{\sum_{i=2}^{L} c_i p_i Z_i + \sigma_N^2}, \quad (8)$$
$$Z_i \sim \chi_2^2, Z_i's \text{ i.i.d.}$$

Then, dividing the top and bottom of the right hand side of Eqn. (8) by $c_1 p_1$ gives

$$\Gamma = \frac{X}{Y + \sigma^2},$$

where $X = Z_1$, $Y = \sum_{i=2}^{L} k_i Z_i$, $k_i = \frac{c_i p_i}{c_1 p_1}$, and $\sigma^2 = \frac{\sigma_N^2}{c_1 p_1}$. With these substitutions, ρ_{out} has the desired form of Eqn. (7). This concludes *Step 1*.

Step 2: This step expresses ρ_{out} in terms of the moment-generating function of Y to obtain the final result. The analysis here uses techniques similar to work done in [19],[20]. Consider

$$\begin{aligned}\rho_{out} &= \Pr(\frac{X}{Y + \sigma^2} \leq \gamma) = \Pr(X \leq \gamma(Y + \sigma^2)) \\ &= \int_0^\infty \Pr(X \leq \gamma(y + \sigma^2)) f_Y(y)\, dy \end{aligned} \quad (9)$$

Here, observe that $Pr(X \leq \gamma(y+\sigma^2))$ is the cumulative distribution function (CDF) of the exponential distribution ($\lambda = \frac{1}{2}$) evaluated at $\gamma(y + \sigma^2)$, which gives

$$Pr(X \leq \gamma(y + \sigma^2)) = 1 - e^{-\frac{\gamma}{2}(y+\sigma^2)}$$

Substituting this expression into Eqn. (9) yields

$$\begin{aligned}\rho_{out} &= \int_0^\infty (1 - e^{-\frac{\gamma}{2}(y+\sigma^2)}) f_Y(y)\, dy \\ &= 1 - e^{-\frac{\gamma}{2}\sigma^2} \int_0^\infty e^{-\frac{\gamma}{2}y} f_Y(y)\, dy \end{aligned} \quad (10)$$

Here, the key observation is that the expression in the integral in Eqn. (10) is the moment-generating function of Y, denoted $\psi_Y(t)$, evaluated at $-\frac{\gamma}{2}$. Since Y is a sum of independent random variables, the moment-generating function of Y is the product of the moment-generating functions of each element in the sum. The moment-generating function of an exponential random variable is given as

$$\psi_{Z_i}(t) = E[e^{tZ_i}] = (1 - 2t)^{-1}$$

The moment-generating function of Y is then given as

$$\begin{aligned}\psi_Y(t) &= E[e^{tY}] = E[e^{t\sum_{i=2}^{L} k_i Z_i}] = \prod_{i=2}^{L} E[e^{tk_i Z_i}] \\ &= \prod_{i=2}^{L} \psi_{Z_i}(k_i t) = \prod_{i=2}^{L} (1 - 2k_i t)^{-1}\end{aligned}$$

Then, evaluating $\psi_Y(t)$ at $t = -\frac{\gamma}{2}$, substituting the result into Eqn. (10), and substituting $\frac{c_i p_i}{c_1 p_1} = k_i$ gives the result:

$$\rho_{out} = 1 - e^{-\frac{\gamma}{2}\sigma^2} \prod_{i=2}^{L} \left(1 + \gamma \frac{c_i p_i}{c_1 p_1}\right)^{-1}$$

■

C. Observations on the Optimization Problem

Using the expression for outage probability above and substituting $\sigma_l = \frac{\sigma_{N_l}^2}{c_{ll} p_l}$, the optimization problem in (3) can be written as

$$\begin{aligned} &\min_{\mathbf{p} \geq 0, \mathbf{U}, \mathbf{V}} \mathbf{w}^T \mathbf{p} \\ &\text{s.t.} \quad 1 - e^{-\frac{\gamma_l}{2}\frac{\sigma_{N_l}^2}{c_{ll} p_l}} \prod_{i \neq l} \left(1 + \gamma_l \frac{c_{li} p_i}{c_{ll} p_l}\right)^{-1} \leq \alpha_l, l = 1, \ldots, L \end{aligned} \quad (11)$$

To get the constraints in (11) into a more convenient form, define

$$g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V}) = e^{\frac{\gamma_l}{2}\frac{\sigma_{N_l}^2}{c_{ll} p_l}} \prod_{i \neq l} \left(1 + \gamma_l \frac{c_{li} p_i}{c_{ll} p_l}\right)$$
$$\beta_l = (1 - \alpha_l)^{-1}$$

Then, with some manipulation on the constraints, (11) can be rewritten as

$$\begin{aligned} &\min_{\mathbf{p}\geq 0,\mathbf{U},\mathbf{V}} \mathbf{w}^T\mathbf{p} \\ &\text{s.t.} \quad g_l(\gamma_l,\mathbf{p},\mathbf{u}_l,\mathbf{V}) \leq \beta_l, l=1,\ldots,L \end{aligned} \tag{12}$$

The form presented in (12) will serve as the main optimization problem used for analysis in the rest of this work. To gain more insight into this problem, two lemmas are given that will aid in the algorithms and manipulations for future sections. These lemmas are extensions of the work in [7] to this problem. The proofs are given in Appendix I.

Lemma 2: Holding the beamformers $\mathbf{U}$ and $\mathbf{V}$ constant, the optimization problem in (12) has the following 3 properties:

1) If it exists, the optimum solution $\mathbf{p}^*$ is unique.
2) All the outage constraints are active (i.e. hold with equality) for $\mathbf{p}=\mathbf{p}^*$.
3) $\mathbf{p}^*$ is unaffected by the choice of $\mathbf{w}$.

Intuitively, all the constraints are active since if any of them held with strict inequality, the objective function could be reduced by making the constraint hold with equality. Also, since all the constraints hold with equality, there are L equations (the constraints) and L unknowns (the p_l's). Thus, the constraints uniquely determine the optimal solution $\mathbf{p}^*$, and so the weighting vector has no effect on the solution. It follows from *Lemma 2* that for fixed transmit and receive beamformers, solving the constraints with equality will yield the optimal power allocation scheme, and the objective function in (12) can be ignored. The next related Lemma, analogous to [7], will aid in gaining insight into suboptimal solutions in Section IV-C.

Lemma 3: Consider the following optimization problem:

$$\begin{aligned} &\max_{\mathbf{p}\geq 0,\mathbf{U},\mathbf{V}} \mathbf{w}^T\mathbf{p} \\ &\text{s.t.} \quad g_l(\gamma_l,\mathbf{p},\mathbf{u}_l,\mathbf{V}) \geq \beta_l, l=1,\ldots,L \end{aligned} \tag{13}$$

This problem has the same solution as the optimization problem in (12).

Intuitively, the optimization problem in (13) is the same as the problem in (12) since once again, all the constraints will be active, so the constraints uniquely determine $\mathbf{p}^*$.

This foundation aids in the development of algorithms to find solutions for the optimization problem in (12). First, the simpler single user MIMO problem is considered.

III. The Single User MIMO Problem

A. Optimal Power Control

Despite the rich literature in single user MIMO, the problem where *both* the transmitter and receiver only know statistical information about the channel and want to achieve a certain SINR threshold with some probability has not, to our knowledge, been studied. Here, there is no interference in the system, and the weighted cost function does not apply since the problem requires minimizing over one variable. Thus, the optimization problem for this special case is as follows:

$$\begin{aligned} &\min_{P\geq 0,\mathbf{u},\mathbf{v}} P \\ &\text{s.t.} \quad \exp\left(\frac{\gamma\sigma_N^2}{2P(\mathbf{v}^*\otimes\mathbf{u})^H\boldsymbol{\Sigma}(\mathbf{v}^*\otimes\mathbf{u})}\right) \leq \beta \end{aligned} \tag{14}$$

Although there are no interference terms, it can be seen that the proof for *Lemma 2* holds, so the constraint in (14) is satisfied by equality. Solving the constraint for P gives

$$P = \frac{\gamma\sigma_N^2}{2(\mathbf{v}^*\otimes\mathbf{u})^H\boldsymbol{\Sigma}(\mathbf{v}^*\otimes\mathbf{u})\log\beta} \tag{15}$$

The main challenge in this problem is then to find the optimal beamformers $\mathbf{u}$ and $\mathbf{v}$. From Eqn. (15), it can be seen that P will be minimized when $(\mathbf{v}^*\otimes\mathbf{u})^H\boldsymbol{\Sigma}(\mathbf{v}^*\otimes\mathbf{u})$ is maximized. Thus, the following optimization problem follows:

$$\begin{aligned} &\max_{\mathbf{u},\mathbf{v}} (\mathbf{v}^*\otimes\mathbf{u})^H\boldsymbol{\Sigma}(\mathbf{v}^*\otimes\mathbf{u}) \\ &\text{s.t.} \quad ||\mathbf{u}||_2=1, ||\mathbf{v}||_2=1 \end{aligned} \tag{16}$$

Solving for the beamformers in (16) is equivalent to solving for the beamformers in (14). First, the Kronecker model will be considered, where a closed-form solution can be obtained. Then, the general correlated Rayleigh fading case is considered, where an iterative algorithm to find the beamformers is derived.

B. Optimal Beamforming for the Kronecker Model

The Kronecker model assumes that the spatial covariance matrix can be broken into the spatial correlation at each link end, so the covariance matrix is given as

$$\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_T^T \otimes \boldsymbol{\Sigma}_R \tag{17}$$

This model is widely used, and its validity is discussed and verified in [15][23][24]. To obtain the beamforming optimization problem for the Kronecker model, substitute the expression for $\boldsymbol{\Sigma}$ given in Eqn. (17) into (16). The objective function in (16) then becomes

$$(\mathbf{v}^* \otimes \mathbf{u})^H \mathbf{\Sigma}(\mathbf{v}^* \otimes \mathbf{u}) = (\mathbf{v}^* \otimes \mathbf{u})^H (\mathbf{\Sigma}_T^T \otimes \mathbf{\Sigma}_R)(\mathbf{v}^* \otimes \mathbf{u})$$
$$= (\mathbf{v}^T \mathbf{\Sigma}_T^T \mathbf{v}^*)(\mathbf{u}^H \mathbf{\Sigma}_R \mathbf{u}) = (\mathbf{v}^H \mathbf{\Sigma}_T \mathbf{v})(\mathbf{u}^H \mathbf{\Sigma}_R \mathbf{u})$$

The second equality follows from the property of Kronecker products that states

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC} \otimes \mathbf{BD}), \tag{18}$$

and the last equality follows from $a^T = a$, where a is a scalar (so $\mathbf{v}^T \mathbf{\Sigma}_T^T \mathbf{v}^* = (\mathbf{v}^T \mathbf{\Sigma}_T^T \mathbf{v}^*)^T = \mathbf{v}^H \mathbf{\Sigma}_T \mathbf{v}$). With these manipulations, the maximization problem in (16) then becomes

$$\begin{aligned} &\max_{\mathbf{u},\mathbf{v}} (\mathbf{v}^H \mathbf{\Sigma}_T \mathbf{v})(\mathbf{u}^H \mathbf{\Sigma}_R \mathbf{u}) \\ &\text{s.t.} \quad ||\mathbf{u}||_2 = 1, ||\mathbf{v}||_2 = 1 \end{aligned} \tag{19}$$

Note that $\mathbf{u}$ and $\mathbf{v}$ are now separated, so two separate maximization problems arise whose solutions are well known: $\mathbf{u}$ is the (normalized) eigenvector that corresponds to the largest eigenvalue of $\mathbf{\Sigma}_R$, and $\mathbf{v}$ is the (normalized) eigenvector that corresponds to the largest eigenvalue of $\mathbf{\Sigma}_T$. It is worth noting that if $\mathbf{\Sigma}_T$ and $\mathbf{\Sigma}_R$ are set to the identity matrices of appropriate dimension, then $\mathbf{\Sigma} = \mathbf{I}$ and the problem simplifies to the special case of i.i.d. Rayleigh fading. In this case, any set of transmit and receive beamformers are optimal, since all vectors are eigenvectors of $\mathbf{I}$, and the eigenvalues are all 1.

After they are calculated, the solutions for $\mathbf{u}$ and $\mathbf{v}$ can be substituted back into the constraint of (14) to solve for P. Define $\lambda_T^{(1)}$ and $\lambda_R^{(1)}$ as the largest eigenvalues of $\mathbf{\Sigma}_T$ and $\mathbf{\Sigma}_R$, respectively. Then P_{opt} is given by

$$P_{opt} = \frac{\gamma \sigma_N^2}{2\lambda_T^{(1)} \lambda_R^{(1)} \log \beta} \tag{20}$$

C. Beamforming for General Rayleigh Fading

Now consider the general Rayleigh fading case. Using the property of Kronecker products given in Eqn. (18), observe that

$$\mathbf{v}^* \otimes \mathbf{u} = (\mathbf{v}^* \otimes \mathbf{I}_{N_R})(1 \otimes \mathbf{u}) = (\mathbf{v}^* \otimes \mathbf{I}_{N_R})\mathbf{u} \tag{21}$$
$$\mathbf{v}^* \otimes \mathbf{u} = (\mathbf{I}_{N_T} \otimes \mathbf{u})(\mathbf{v}^* \otimes 1) = (\mathbf{I}_{N_T} \otimes \mathbf{u})\mathbf{v}^* \tag{22}$$

Then, define

$$\begin{aligned} \mathbf{\Sigma}_\mathbf{v} &= (\mathbf{v}^* \otimes \mathbf{I}_{N_R})^H \mathbf{\Sigma} (\mathbf{v}^* \otimes \mathbf{I}_{N_R}) \\ \mathbf{\Sigma}_\mathbf{u} &= (\mathbf{I}_{N_T} \otimes \mathbf{u})^H \mathbf{\Sigma} (\mathbf{I}_{N_T} \otimes \mathbf{u}) \end{aligned}$$

From Eqns. (21) and (22), it follows that

$$(\mathbf{v}^* \otimes \mathbf{u})^H \mathbf{\Sigma}(\mathbf{v}^* \otimes \mathbf{u}) = \mathbf{v}^T \mathbf{\Sigma}_\mathbf{u} \mathbf{v}^* = \mathbf{u}^H \mathbf{\Sigma}_\mathbf{v} \mathbf{u}$$

It can be shown that $\mathbf{\Sigma}_\mathbf{u}$ and $\mathbf{\Sigma}_\mathbf{v}$ are positive semidefinite matrices since for a positive semidefinite matrix $\mathbf{\Sigma}$, $\mathbf{A}^H \mathbf{\Sigma} \mathbf{A}$ is also positive semidefinite [25]. Now, $\mathbf{\Sigma}_\mathbf{u}$ can be used to solve for the optimal transmit beamformer for a fixed receive beamformer, and $\mathbf{\Sigma}_\mathbf{v}$ can be used to solve for the optimal receive beamformer for a fixed transmit beamformer. In this way, an iterative algorithm can be derived to obtain both the transmit and receive beamformers. First, if $\mathbf{v}$ is fixed such that $\mathbf{\Sigma}_\mathbf{v} \succeq 0$, with at least 1 nonzero eigenvalue (random initialization should suffice), the optimal solution for $\mathbf{u}$ will be the eigenvector corresponding to the largest eigenvalue of $\mathbf{\Sigma}_\mathbf{v}$. Then, if $\mathbf{u}$ is then fixed, the optimal solution for $\mathbf{v}$ will be the complex conjugate of the eigenvector corresponding to the largest eigenvalue of $\mathbf{\Sigma}_\mathbf{u}$. This algorithm can alternately solve for $\mathbf{u}$ while fixing $\mathbf{v}$ and then solve for $\mathbf{v}$ while fixing $\mathbf{u}$. The optimization problem in (16) is not jointly convex in $\mathbf{u}$ and $\mathbf{v}$, so this algorithm will converge monotonically to a locally optimal solution. To initialize the algorithm, the Kronecker model approximation can be used on the covariance matrix, and the optimal beamformers from this approximation can be used. While this solution is not proven to be globally optimal, it is shown in Section V to perform well.

Once the algorithm to obtain the transmit and receive beamformers is applied, a solution for P can be obtained using Eqn. (15):

$$P_{opt} = \frac{\gamma \sigma_N^2}{2(\mathbf{v}_{opt}^* \otimes \mathbf{u}_{opt})^H \mathbf{\Sigma} (\mathbf{v}_{opt}^* \otimes \mathbf{u}_{opt}) \log \beta} \tag{23}$$

IV. The Multiuser MIMO Problem

A. Optimal Power Control

Now return to the multiuser problem in Eqn. (12). This problem is more difficult to solve than the single-user case due to interference from other users. In order to make the problem tractable, first examine the problem when both the transmit and receive beamformers are fixed. With this assumption, a power control algorithm solving the constraint equations in (12) with equality is derived. Algorithms for the beamformers are then studied in the following sections.

To derive an optimal power control algorithm, start by taking the logarithm of the constraint equations in (12):

$$\log(g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V})) = \log\left(e^{\frac{\gamma_l}{2}\frac{\sigma_{N_l}^2}{c_{ll}p_l}}\prod_{i\neq l}\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right)\right)$$
$$= \frac{\gamma_l\sigma_{N_l}^2}{2c_{ll}p_l} + \sum_{i\neq l}\log\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right)$$
$$\leq \log\beta_l$$

Multiplying both sides by $\frac{p_l}{\log\beta_l}$ results in

$$\frac{\gamma_l\sigma_{N_l}^2}{2c_{ll}\log\beta_l} + \frac{p_l}{\log\beta_l}\sum_{i\neq l}\log\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right) \leq p_l \quad (24)$$

Now define

$$I_l(\mathbf{p}) = \frac{\gamma_l\sigma_{N_l}^2}{2c_{ll}\log\beta_l} + \frac{p_l}{\log\beta_l}\sum_{i\neq l}\log\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right)$$
$$\mathbf{I}(\mathbf{p}) = [I_1(\mathbf{p}), \ldots, I_L(\mathbf{p})]$$

The function $\mathbf{I}(\mathbf{p})$ is a *standard interference function*. For the definition of standard interference functions and the proof, see Appendix II. A key property of standard interference functions is that they satisfy $\mathbf{I}(\mathbf{p}) \leq \mathbf{p}$. Thus, using this function and a starting power vector $\mathbf{p}$, the update equation for the algorithm is given as:

$$\mathbf{p}^{(n+1)} = \mathbf{I}(\mathbf{p}^{(n)}) \quad (25)$$

The algorithm is then given as follows:

Algorithm 1: Power Control

1) Initialize $\mathbf{p} \geq 0$, $\mathbf{U}$, and $\mathbf{V}$
2) Update $\mathbf{p}$ using (25) until convergence.

This algorithm will be used jointly with beamforming algorithms in future sections to obtain the optimal power allocation for a given set beamformers.

B. Optimal Beamforming for the Kronecker Model

While for general Rayleigh fading the optimal transmit and receive beamformers are difficult to obtain, the problem in (12) can be solved if the Kronecker model is assumed on all the links. The Kronecker model assumes the transmit and receive correlations are independent at the link ends. *This implies that each transmitter has its own transmit correlation matrix, regardless of the receiver, and each receiver has its own receive correlation matrix, regardless of the transmitter.* Thus, the channel from transmitter i to receiver l is given as:

$$\text{vec}(\mathbf{H}_{li}) \sim CN(0, \mathbf{\Sigma}_{T_i}^T \otimes \mathbf{\Sigma}_{R_l})$$

This assumption has some interesting implications. Note that now, for the Kronecker model, the link gain between receiver l and transmitter i is given by

$$c_{li} = (\mathbf{v}_i^* \otimes \mathbf{u}_l)^H(\mathbf{\Sigma}_{T_i}^T \otimes \mathbf{\Sigma}_{R_l})(\mathbf{v}_i^* \otimes \mathbf{u}_l)$$
$$= (\mathbf{v}_i^T\mathbf{\Sigma}_{T_i}^T\mathbf{v}_i^*)(\mathbf{u}_l^H\mathbf{\Sigma}_{R_l}\mathbf{u}_l) = (\mathbf{v}_i^H\mathbf{\Sigma}_{T_i}\mathbf{v}_i)(\mathbf{u}_l^H\mathbf{\Sigma}_{R_l}\mathbf{u}_l)$$

The last equality follows from $a^T = a$, where a is a scalar (as in the single-user case above). The constraint equations in (12) then become

$$\exp\left(\frac{\gamma_l}{2}\frac{\sigma_{N_l}^2}{(\mathbf{v}_l\mathbf{\Sigma}_{T_l}\mathbf{v}_l)(\mathbf{u}_l^H\mathbf{\Sigma}_{R_l}\mathbf{u}_l)p_l}\right)\times$$
$$\prod_{i\neq l}\left(1+\gamma_l\frac{(\mathbf{v}_i\mathbf{\Sigma}_{T_i}\mathbf{v}_i)p_i}{(\mathbf{v}_l\mathbf{\Sigma}_{T_l}\mathbf{v}_l)p_l}\right) \leq \beta_l, l = 1, \ldots, L \quad (26)$$

First, focus on the receive beamformers. The important observation here is that $\mathbf{u}_l$ only appears in the exponential noise term in the product in Eqn. (26), and cancels out in the other product terms, which affect the interference. Therefore, *the receive beamforming only helps to mitigate the noise, and not the interference*. With respect to the receive beamforming, then, this case is exactly analogous to the single user MIMO case, and the optimal $\mathbf{u}_l$ is given by the normalized eigenvector corresponding to the largest eigenvalue of $\mathbf{\Sigma}_{R_l}, l = 1, \ldots, L$. From this solution, note that $\mathbf{u}_{l_{opt}}^H\mathbf{\Sigma}_{R_l}\mathbf{u}_{l_{opt}} = \lambda_{R_l}^{(1)}$, where $\lambda_{R_l}^{(1)}$ is the maximum eigenvalue of $\mathbf{\Sigma}_{R_l}$.

For the optimal transmit beamformers, define $q_l = (\mathbf{v}_l^H\mathbf{\Sigma}_{T_l}\mathbf{v}_l)p_l, l = 1, \ldots, L$. By noting that $p_l = \frac{q_l}{(\mathbf{v}_l^H\mathbf{\Sigma}_{T_l}\mathbf{v}_l)}$, assuming the Kronecker model, the following equivalent optimization problem to (12) is presented:

$$\min_{\mathbf{q}\geq 0, \mathbf{U}, \mathbf{V}} \sum_{l=1}^{L}\frac{w_l q_l}{(\mathbf{v}_l^H\mathbf{\Sigma}_{T_l}\mathbf{v}_l)}$$
$$\text{s.t.} \quad \exp\left(\frac{\gamma_l\sigma_{N_l}^2}{2\lambda_{R_l}^{(1)}q_l}\right)\prod_{i\neq l}\left(1+\gamma_l\frac{q_i}{q_l}\right) \leq \beta_l, l = 1, \ldots, L \quad (27)$$

In the optimization problem in (27), the $\mathbf{v}_l$'s only appear in the optimization function, and not in the constraints. To minimize the objective function in (27), then, the denominator terms should be maximized. Once again, this optimization problem is analogous to the single-user case, and the optimal $\mathbf{v}_l$'s are given by the normalized eigenvectors corresponding to the maximum eigenvalues of the $\mathbf{\Sigma}_{T_l}$'s. Since the problem in (27) is equivalent to the problem in (12) for the Kronecker model, the optimal transmit beamformers will be the same as well.

For the special case of i.i.d. Rayleigh fading on all the links, e.g. when $\mathbf{\Sigma}_{li} = \mathbf{I}\,\forall l, i$, it follows from this solution that any set of transmit and receive beamformers is optimal, using the same arguments as the single-user case. After substituting the optimal transmit and receive beamformers into (12), **Algorithm 1** can be run to solve for the optimal power allocation, yielding the jointly optimal solution.

Note that due to the limitations of the Kronecker model, the optimal transmit and receive beamformers are not able to intelligently suppress the interference from other users. This is not the case for the general Rayleigh fading case, as is shown in the next section.

C. A Near-Optimal Solution for General Rayleigh Fading

The Kronecker model proved to be analytically tractable, but the assumptions made by this model may not be adequate for many multiuser setups–the transmitter location will affect both the scatterers that are used around both the receiver and transmitter in many scenarios. Both receive and transmit beamforming for the general Rayleigh fading case, however, are more difficult to solve. As such, a suboptimal algorithm is considered in this section, using bounds on the constraints to find the optimal solution for the ratio of the *average* of the signal to the *average* of the interference plus noise. Looking at this framework allows use of modified versions of previously developed techniques for algorithms utilizing perfect channel knowledge. This work builds off of [7], adapting their framework for the optimization problem in (12).

1) Problem Setup: First, consider the bounds given in [21],[22] for the outage probability, adapted to the constraints in (12):

Lemma 4: Upper and lower bounds for $g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V})$ are given by

$$1+\gamma_l \frac{\sum_{i\neq l} c_{li}p_i + \frac{1}{2}\sigma_{N_l}^2}{c_{ll}p_l} \leq g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V}) \leq \exp\left\{\frac{\gamma_l(\sum_{i\neq l} c_{li}p_i + \frac{1}{2}\sigma_{N_l}^2)}{c_{ll}p_l}\right\}$$

The proof is given in Appendix I. First, consider using the expression for the upper bound instead of $g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V})$ in the constraints of (12):

$$\begin{aligned} &\min_{\mathbf{p}\geq 0, \mathbf{u}_l, \mathbf{v}_l} \mathbf{w}^T\mathbf{p} \\ \text{s.t.} \quad &\exp\left\{\gamma_l \frac{\sum_{i\neq l} c_{li}p_i + \frac{1}{2}\sigma_{N_l}^2}{c_{ll}p_l}\right\} \leq \beta_l, l = 1, \ldots, L \end{aligned} \tag{28}$$

The solutions to the optimization problem in (28) will yield feasible solutions to the problem in (12), since the upper bound will never be less than $g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V})$. The the constraint equations can then be rearranged to get

$$\begin{aligned} &\min_{\mathbf{p}\geq 0, \mathbf{U}, \mathbf{V}} \mathbf{w}^T\mathbf{p} \\ \text{s.t.} \quad &\frac{c_{ll}p_l}{\sum_{i\neq l} c_{li}p_i + \frac{1}{2}\sigma_{N_l}^2} \geq \frac{\gamma_l}{\log \beta_l}, l = 1, \ldots, L \end{aligned} \tag{29}$$

The lower bound can also be used for optimization. Using *Lemma 3*, consider the optimization problem from (13). Substituting in the lower bound from *Lemma 4* for $g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V})$ in the constraint equations, after some manipulation, results in the following optimization problem:

$$\begin{aligned} &\max_{\mathbf{p}\geq 0, \mathbf{U}, \mathbf{V}} \mathbf{w}^T\mathbf{p} \\ \text{s.t.} \quad &\frac{c_{ll}p_l}{\sum_{i\neq l} c_{li}p_i + \frac{1}{2}\sigma_{N_l}^2} \leq \frac{\gamma_l}{\beta_l - 1}, l = 1, \ldots, L \end{aligned}$$

This problem can also be stated as [7]

$$\begin{aligned} &\min_{\mathbf{p}\geq 0, \mathbf{U}, \mathbf{V}} \mathbf{w}^T\mathbf{p} \\ \text{s.t.} \quad &\frac{c_{ll}p_l}{\sum_{i\neq l} c_{li}p_i + \frac{1}{2}\sigma_{N_l}^2} \geq \frac{\gamma_l}{\beta_l - 1}, l = 1, \ldots, L \end{aligned} \tag{30}$$

The solutions to the problem in (30) will not yield strictly feasible solutions to (12), but **Algorithm 1** can be applied to the obtained solution to achieve a feasible solution for (12).

In most networks, the outage probability thresholds will be small to increase the likelihood that the links are active. If the outage probability threshold α_l is small, then $\beta_l = (1 - \alpha_l)^{-1}$ is larger than, but very close to 1. Then, from the approximation $\log(1+x) \approx x$, it follows that $\log \beta_l \approx \beta_l - 1$, or

$$\frac{\gamma_l}{\log \beta_l} \approx \frac{\gamma_l}{\beta_l - 1}$$

Thus, from (29) and (30), the upper and lower bounds are tight for low outage probabilities.

Also, note that the optimization problems in (29) and (30) take *exactly* the same form as the optimization problems from [7] (only certain constants are different). Thus all the techniques from this work can be applied to solve for the powers and the transmit and receive beamformers. Note that while the problems look similar mathematically, the quantities in the problem presented in [7] and the problem presented here vary significantly–the problem in [7] utilizes CSI (thus, knowledge of $\{\mathbf{H}_{li}\}$), while the problem presented here utilizes CDI (knowledge of $\{\mathbf{\Sigma}_{li}\}$).

2) Optimal Power Control and Beamforming for the Bounded Problem: For now, consider the optimization problem in (29). It can be shown that all the techniques discussed here can also be used for the optimization problem in (30). For notational convenience, first define

$$\begin{aligned} n_l &= \frac{1}{2}\sigma_{N_l}^2 \\ \psi_l &= \frac{\gamma_l}{\log \beta_l} \\ \Psi_l &= \frac{c_{ll}p_l}{\sum_{i\neq l} c_{li}p_i + n_l} \end{aligned}$$

Then (29) can be rewritten as

$$\begin{aligned} &\min_{\mathbf{p}\geq 0,\mathbf{U},\mathbf{V}} \mathbf{w}^T\mathbf{p} \\ &\text{s.t.} \quad \Psi_l \geq \psi_l, l = 1,\ldots,L \end{aligned} \tag{31}$$

The optimization problem in (31) can be rewritten as a linear programming problem [7]. First, define an L by L diagonal matrix $\mathbf{D}$, with elements $\mathbf{D}_{ll} = \frac{c_{ll}}{\psi_l}$, and a L by L matrix $\mathbf{C}_I$ with c_{li} as its elements on the off-diagonal and 0's along the diagonal. Using these two matrices, also define $\mathbf{A} = \mathbf{D} - \mathbf{C}_I$. After some manipulation, the problem in (31) can be rewritten as:

$$\begin{aligned} &\min_{\mathbf{p}\geq 0,\mathbf{U},\mathbf{V}} \mathbf{w}^T\mathbf{p} \\ &\text{s.t.} \quad \mathbf{A}\mathbf{p} \geq \mathbf{n} \end{aligned} \tag{32}$$

The suboptimal solution will have a feasible solution when $\mathbf{D}^{-1}\mathbf{C}_I$ has a spectral radius less than 1 [7]. It can be shown that if a feasible solution exists, the constraint inequality in (32) is satisfied with equality, so $\mathbf{p}$ is given as:

$$\mathbf{p} = \mathbf{A}^{-1}\mathbf{n}$$

Also, the receive beamformers can be solved in this setup for fixed transmit beamformers and powers. To minimize the objective function, $\Psi_l, l = 1,\ldots,L$, should be maximized for $\mathbf{U}$. Doing this allows for $\mathbf{p}$ (specifically, p_l for the l^{th} constraint) to be minimized as much as possible while meeting the constraints. Define

$$\begin{aligned} \mathbf{\Phi}_l^{(n)} &= p_l(\mathbf{v}_l^* \otimes \mathbf{I})^H \mathbf{\Sigma}_{ll}(\mathbf{v}_l^* \otimes \mathbf{I}) \\ \mathbf{\Phi}_l^{(d)} &= \sum_{i\neq l} p_i(\mathbf{v}_i^* \otimes \mathbf{I})^H \mathbf{\Sigma}_{li}(\mathbf{v}_i^* \otimes \mathbf{I}) + n_l\mathbf{I} \end{aligned}$$

The following L optimization problems can then be written:

$$\begin{aligned} \max_{\mathbf{u}_l} \Psi_l &= \frac{c_{ll}p_l}{\sum_{i\neq l} c_{li}p_i + n_l} \\ &= \frac{p_l(\mathbf{v}_l^* \otimes \mathbf{u}_l)^H \mathbf{\Sigma}_{ll}(\mathbf{v}_l^* \otimes \mathbf{u}_l)}{\sum_{i\neq l} p_i(\mathbf{v}_i^* \otimes \mathbf{u}_l)^H \mathbf{\Sigma}_{li}(\mathbf{v}_i^* \otimes \mathbf{u}_l) + n_l} \\ &= \frac{\mathbf{u}_l^H \mathbf{\Phi}_l^{(n)} \mathbf{u}_l}{\mathbf{u}_l^H \mathbf{\Phi}_l^{(d)} \mathbf{u}_l}, l = 1,\ldots,L \end{aligned} \tag{33}$$

Note that Ψ_l is only dependent on the l^{th} receive beamformer, so the constraints can be optimized individually. Here, the optimal $\mathbf{U}$ takes a slightly different form than the optimal $\mathbf{U}$ in [7], where the optimal receive beamformers are given by the MVDR beamformers. In this problem, the above equation takes the form of the dominant eigenvector for the generalized eigenvalue/eigenvector problem, which is well-studied [26]. Specifically, $\mathbf{u}_l$ is the normalized vector that satisfies the following equation for the largest possible λ:

$$\mathbf{\Phi}_l^{(n)}\mathbf{x} = \lambda \mathbf{\Phi}_l^{(d)}\mathbf{x}$$

Now, focus on the transmit beamformers. The transmit beamformers can be solved in an analogous fashion to the receive beamformers for the dual optimization problem of (32). It can be shown that this dual optimization problem can be formulated as

$$\begin{aligned} &\min_{\mathbf{q}\geq 0,\mathbf{U},\mathbf{V}} \mathbf{n}^T\mathbf{q} \\ &\text{s.t.} \quad \mathbf{A}^T\mathbf{q} \geq \mathbf{w} \end{aligned} \tag{34}$$

Once again, using the same arguments as before, the constraint inequality in (34) is satisfied with equality. Thus, the solution to the optimum power allocation in the dual domain is given by

$$\mathbf{q} = \mathbf{A}^{-T}\mathbf{w}$$

Then, the optimal transmit beamformers can be solved for by noticing that the l^{th} constraint in dual problem is only dependent on the l^{th} transmit beamformer (all else held constant), for fixed receive beamformers and a fixed dual power vector $\mathbf{q}$. Define for the dual problem

$$\begin{aligned} \tilde{\mathbf{\Phi}}_l^{(n)} &= q_l(\mathbf{I} \otimes \mathbf{u}_l)^H \mathbf{\Sigma}_{ll}(\mathbf{I} \otimes \mathbf{u}_l) \\ \tilde{\mathbf{\Phi}}_l^{(d)} &= \sum_{i\neq l} q_i(\mathbf{I} \otimes \mathbf{u}_i)^H \mathbf{\Sigma}_{li}(\mathbf{I} \otimes \mathbf{u}_i) + w_l\mathbf{I} \end{aligned}$$

This problem is analogous to the receive beamformer problem, so the transmit beamformers can be solved as the principal generalized eigenvector for $\tilde{\mathbf{\Phi}}_l^{(n)}$ and $\tilde{\mathbf{\Phi}}_l^{(d)}$.

From the above arguments, an algorithm for joint power control and transmit/receive beamforming can be derived:

Algorithm 2: Joint Power Control and Beamforming Algorithm

1) Initialize $\mathbf{p} \geq 0$ and $\mathbf{V}$
2) Calculate the $\mathbf{\Phi}_l$'s and use them to calculate $\mathbf{U}$
3) Calculate $\mathbf{A}$, and then calculate $\mathbf{q} = \mathbf{A}^{-T}\mathbf{w}$
4) In the dual domain, calculate the $\tilde{\mathbf{\Phi}}_l$'s and use them to calculate $\mathbf{V}$
5) Calculate $\mathbf{A}$, and then calculate $\mathbf{p} = \mathbf{A}^{-1}\mathbf{n}$
6) Repeat steps 2-5 until convergence.

This algorithm converges regardless of the initialization and will generate feasible solutions of decreasing cost [7]. One way to initialize the algorithm is to ignore interference and use the algorithms for point-to-point communication given in section III.

After **Algorithm 2** has converged, **Algorithm 1** can also be used to achieve the optimal power control scheme for the beamformers produced by this algorithm. To more closely approach the optimal solution, the following algorithm can be used:

Algorithm 3: Extensive Joint Power Control and Beamforming Algorithm

1) Initialize $\mathbf{p} \geq 0$ and $\mathbf{V}$
2) Run **Algorithm 2** on (29), and run **Algorithm 1** on the result
3) Run **Algorithm 2** on (30), and run **Algorithm 1** on the result
4) Compare steps 2 and 3 and take the set of beamformers and power vectors for initialization in step 1
5) Repeat steps 1-4 until convergence.

While **Algorithm 3** will always perform at least as well as **Algorithm 2** on its own, the results from the next section show this algorithm only provides marginal improvement. **Algorithm 2** requires less run-time, however, since **Algorithm 3** requires at least 2 runs of **Algorithm 2**. **Algorithm 2** is also on the same order of complexity as the CSI algorithm presented in [7]. However, since CDI changes much less frequently than CSI, algorithms based on CDI require less feedback and need to be run much less frequently than their CSI counterparts.

V. Results

A. Simulation Parameters

We conduct some numerical experiments to understand the efficacy of the joint power control and beamforming algorithms developed above. Results are given for a SU-MIMO setup and a MU-MIMO setup. For the covariance matrices in both cases, an angular spread model with 100 scatterers and varying transmit and receive angular spreads across the links is considered. The signal links are centered at broadside with transmit and receive angular spreads varying from 5 to 20 degrees, and the interfering links are centered at incident angles with transmit and receive angular spreads varying from 10 to 40 degrees. The parameters were selected with respect to typical scenarios in the 3GPP model [27]. All transmitters and receivers have 4 antennas, and the signal-to-noise ratio (SNR) is held constant at 10 dB. Equal SINR thresholds and equal outage constraints is considered for all users in the system.

For the MU-MIMO setup, a network with 3 transmit/receive links and a weighting vector $\mathbf{w} = [10, 1, 1]^T$ is considered.

B. Single-User MIMO

In *Experiment 1*, the single-user MIMO case is considered. Here, an algorithm using perfect channel knowledge is compared to the algorithm presented in Section III, which uses the covariance information only. The algorithm using perfect channel knowledge uses the principal left and right singular vectors for receiver and transmitter beamforming, respectively, and calculates the minimum transmit power required to achieve the threshold (this is an optimal scheme). The outage value at which the average power required using CSI matches the power required using CDI is a function of the inverse of the expected dominant singular value of $\mathbf{H}$, and can be empirically calculated for a given covariance matrix. See Fig. 1.

The plot yields some interesting results: the average transmit power required for CDI is comparable to the average power required for CSI for the same threshold, with the value being lower at 20% outage. To calculate the CDI powers, only the covariance information is required, so it needs to be updated only when the channel statistics change. On average, 2.5dB less power is required to transmit using CDI at 20% outage as compared to CSI. The tradeoff is that the covariance algorithm will allow the link to fail with the specified outage probability. However, this may be a desirable tradeoff for many network applications since it prevents the transmitter from attempting to increase the power to compensate for poor channel conditions, thereby decreasing the interference to other nodes in the network.

C. Multi-User MIMO

For the multiuser case, the algorithm from [7] is used to calculate the optimal transmit/receive beamformers and powers at each channel instance (using perfect channel knowledge). In *Experiment 2*, the average value of the objective function using the weighting vector above is compared against the suboptimal **Algorithm 2** for varying outage constraints. See Fig. 2.

This experiment shows similar results to the single-user case, except the solutions generated by **Algorithm**

2 become infeasible (when the curves go to ∞). The SINR thresholds here are reasonable, as many cellular networks today are able to operate in the low SINR regime [27]. The lower the outage, the lower the SINR threshold has to be for the outage to become infeasible. Once again, the value of the average objective function is comparable to using true channel knowledge for low enough SINR thresholds. This means that knowing only covariance information, if the system is allowed to fall into outage with some probability, transmission can be performed on average with comparable power than with knowing perfect channel information all the time. Using **Algorithm 2**, there is about a 1dB savings in the average cost function at an SINR threshold of -4dB at 20% outage compared to the CSI scheme.

For more insight into the power consumption using perfect channel knowledge and covariance information, also observe the CDF of the weighted sum power for true channel knowledge at an SINR threshold of 0 dB in Fig. 3. This plot shows what percent of the time the objective function is larger for perfect channel knowledge as compared to the outage curves. The outage curves are just step functions, since for fixed covariance information, they will always have the same value for their objective function. Since the CDF curves for perfect channel knowledge and 20% outage intersect at roughly 80%, the data shows that roughly 20% of the time, using perfect channel knowledge, the objective function is higher than **Algorithm 2** for 20% outage. Also observe that the median (where the CDF takes a value of 0.5) is approximately -23 dB on the CDF plot. The mean from Fig. 2, however, is at approximately -17 dB, a difference of 6 dB, or a factor of about 4. The large disparity between the median and the mean suggest that the data for perfect channel knowledge is skewed when the channel is bad. Thus, using **Algorithm 2** significantly reduces the peak-to-average power ratio for the transmitters (the peak-to-average power ratio is 1 using **Algorithm 2** since the power allocation does not change with the channel).

Experiment 3 shows the the difference between using **Algorithm 2** and **Algorithm 3**. See Fig. 4. The curves show that the objective function is nearly identical with and without optimal power control, and **Algorithm 3** only begins to give any discernible gains when they approach infeasible SINR thresholds. This is consistent with the analysis of the tightness of the bounds in Section IV-C.

Experiment 4 tests the convergence speed of **Algorithm 2**. See Fig. 5. For the setup in the experiment, both the outage values as well as the objective function values converge within 5 iterations to their fixed point solutions. This confirms that this algorithm is nearly identical in computational complexity to the algorithm used for perfect channel knowledge [7]. The big difference here, however, is that **Algorithm 2** needs to be run much less frequently than the algorithm using perfect channel knowledge.

VI. Conclusion

In this work, a framework has been presented for analyzing MIMO beamforming networks when only the channel statistics are known, and algorithms have been derived that give good performance. Optimal algorithms have also been presented for the single user MIMO case, as well as for the multiuser MIMO Kronecker case. A suboptimal algorithm using duality for joint power control and beamforming for the general Rayleigh fading case was also derived. While finding the optimal beamformers for this problem is an open area of research, given the tightness of the bounds and the negligible improvement of using the optimal power control algorithm on top of the presented suboptimal algorithm, the gains in finding the optimal beamformers will likely be negligible for reasonable outage constraints.

When comparing the algorithms presented here to algorithms where the channel is perfectly known, it has been shown that using covariance information can give gains in the cost function of interest. Thus, the solution presented here has two substantial advantages: the computational complexity for the algorithm and feedback information required for covariance information is drastically reduced since both must be done only once for valid covariance information, and transmission power can be saved in the network if a sufficient level of outage is acceptable on the links. Also note here that the algorithms presented here can be extended to decentralized algorithms as proposed in [7], among others, in a straightforward manner, though this is still an open area of research. Knowledge of covariance information has many benefits, and it can be an effective way to reduce feedback in MIMO networks.

Appendix I
Proofs of Lemmas

Proof of Lemma 1: First, consider $Z = \mathbf{u}^H\mathbf{H}\mathbf{v}$. Z is a linear combination of 0-mean complex circular Gaussians, so Z is 0-mean complex circular Gaussian. The next step is to find the variance. To do this, perform the following calculations:

$$var(Z) = E[Z \cdot Z^*] = E[(\mathbf{u}^H \mathbf{H} \mathbf{v})(\mathbf{u}^H \mathbf{H} \mathbf{v})^*] \quad (35)$$
$$= E[\text{vec}(\mathbf{u}^H \mathbf{H} \mathbf{v})\text{vec}(\mathbf{u}^H \mathbf{H} \mathbf{v})^H] \quad (36)$$
$$= E[(\mathbf{v}^T \otimes \mathbf{u}^H)\text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^H(\mathbf{v}^* \otimes \mathbf{u})] \quad (37)$$
$$= (\mathbf{v}^* \otimes \mathbf{u})^H \mathbf{\Sigma} (\mathbf{v}^* \otimes \mathbf{u}) \quad (38)$$

The key observation here is that equality (36) follows from observing that $x = \text{vec}(x)$ if x is a scalar. Furthermore, for scalars, the Hermitian and conjugate operations are interchangeable. Equality (37) follows from the property that $\text{vec}(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})\text{vec}(\mathbf{X})$. Finally, equality (38) follows from separating out the deterministic factors from the expectation operator and noting that by definition, $E[\text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^H] = \mathbf{\Sigma}$. This gives an expression for $var(Z)$.

The norm squared of a unit variance complex circular Gaussian has a χ_2^2, or exponential, distribution. Then $|Z|^2$, normalized by $var(Z)$, gives

$$\frac{|Z|^2}{var(Z)} = \frac{|\mathbf{u}^H \mathbf{H} \mathbf{v}|^2}{(\mathbf{v}^* \otimes \mathbf{u})^H \mathbf{\Sigma} (\mathbf{v}^* \otimes \mathbf{u})} \sim \chi_2^2$$

□

Proof of Lemma 2: First, use contradiction to prove 2). Note that $g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V})$ in (12) is a *decreasing* function of p_l. Now, assume that for the l^{th} constraint, $g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V})$ is *strictly* less than β_l in the optimal solution, and all the other inequalities are satisfied. This means p_l can be reduced appropriately to get equality and reduce the value of the objective function. Furthermore, since $g_i(\gamma_i, \mathbf{p}, \mathbf{u}_i, \mathbf{V})$ is an *increasing* function in $p_i, i \neq l$, when p_l is reduced all of the other constraints are still satisfied. Thus, having the l^{th} constraint not satisfy equality contradicts optimality, so all constraints are satisfied with equality.

Also note here that if only the l^{th} constraint holds with strict inequality and the rest of the constraints hold with equality, when p_l is reduced, all of the other constraints become inequalities (since $g_i(\gamma_i, \mathbf{p}, \mathbf{u}_i, \mathbf{V})$ is an function in $p_i, i \neq l$). Thus, the rest of the p_i's, $i \neq l$, can be reduced, which will lead to a further reduction of p_l, until a stable solution is reached that gives equality on all the constraints.

Contradiction can also be used to prove 1). Assume there exists two distinct optimal solutions that minimize the cost function, $\hat{\mathbf{p}}$ and $\mathbf{p}^*$. Then, define k and a such that $a = \max_l \hat{p}_l / p_l^*$ and $k = \text{argmax}\, \hat{p}_l / p_l^*$. Since $\hat{\mathbf{p}}$ and $\mathbf{p}^*$ are distinct and both minimize the cost function (so $\mathbf{w}^T \hat{\mathbf{p}} = \mathbf{w}^T \mathbf{p}^*$), at least one element in $\hat{\mathbf{p}}$ is greater than its corresponding element in $\mathbf{p}^*$ (it also follows that at least one element in $\hat{\mathbf{p}}$ is less than its corresponding element in $\mathbf{p}^*$). Therefore, $a > 1$. Then, using 2) from this lemma, consider the k^{th} constraint, satisfied with equality:

$$\beta_k = g_k(\gamma_k, \mathbf{p}^*, \mathbf{u}_k, \mathbf{V}) = e^{\frac{\gamma_k}{2} \frac{\sigma_{N_k}^2}{c_{kk} p_k^*}} \prod_{i \neq k} \left(1 + \gamma_k \frac{c_{ki} p_i^*}{c_{kk} p_k^*}\right) \quad (39)$$
$$> e^{\frac{\gamma_k}{2} \frac{\sigma_{N_k}^2}{c_{kk} p_k^* a}} \prod_{i \neq k} \left(1 + \gamma_k \frac{c_{ki} p_i^*}{c_{kk} p_k^*}\right) \quad (40)$$
$$= e^{\frac{\gamma_k}{2} \frac{\sigma_{N_k}^2}{c_{kk} p_k^* a}} \prod_{i \neq k} \left(1 + \gamma_k \frac{c_{ki} p_i^* a}{c_{kk} p_k^* a}\right) \quad (41)$$
$$= e^{\frac{\gamma_k}{2} \frac{\sigma_{N_k}^2}{c_{kk} \hat{p}_k}} \prod_{i \neq k} \left(1 + \gamma_k \frac{c_{ki} p_i^* a}{c_{kk} \hat{p}_k}\right) \quad (42)$$
$$\geq e^{\frac{\gamma_k}{2} \frac{\sigma_{N_k}^2}{c_{kk} \hat{p}_k}} \prod_{i \neq k} \left(1 + \gamma_k \frac{c_{ki} \hat{p}_i}{c_{kk} \hat{p}_k}\right) \quad (43)$$
$$= g_k(\gamma_k, \hat{\mathbf{p}}, \mathbf{u}_k, \mathbf{V}) = \beta_k \quad (44)$$

The inequality in (42) comes from the fact that $p_i^* a \geq \hat{p}_i$ for all i because of how a was selected, and that $g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V})$ is an increasing function in $p_i, i \neq l$. Because of the inequality in (40), the above steps show a contradiction, so $\mathbf{p}^*$ must be unique.

There exists a unique solution for L equations, so the constraints completely determine $\mathbf{p}^*$, regardless of $\mathbf{w}$. Thus, 3) follows immediately from 1) and 2), which concludes the proof.

□

Proof of Lemma 3: Similar arguments to the ones used to prove *Lemma 2* can be used for this lemma as well. All the constraint equations will hold with equality by the same argument as given previously, and the solution is unique, being solely determined by the constraints.

□

Proof of Lemma 4: First, consider the lower bound. A well-known bound is $\log(1+x) \leq x$, or $1 + x \leq e^x$. Applying this bound results in

$$\exp\left\{\frac{\gamma_l \sigma_{N_l}^2}{2 c_{ll} p_l}\right\} \geq 1 + \frac{\gamma_l \sigma_{N_l}^2}{2 c_{ll} p_l}$$

Then consider

$$g_l(\gamma_l, \mathbf{p}, \mathbf{u}_l, \mathbf{V}) = e^{\frac{\gamma_l}{2} \frac{\sigma_{N_l}^2}{c_{ll} p_l}} \prod_{i \neq l} \left(1 + \gamma_l \frac{c_{li} p_i}{c_{ll} p_l}\right)$$
$$\geq \left(1 + \frac{\gamma_l \sigma_{N_l}^2}{2 c_{ll} p_l}\right) \prod_{i \neq l} \left(1 + \gamma_l \frac{c_{li} p_i}{c_{ll} p_l}\right) \quad (45)$$

The expression resulting in (45) can be expanded:

$$\left(1+\frac{\gamma_l\sigma_{N_l}^2}{2c_{ll}p_l}\right)\prod_{i\neq l}(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l})$$

$$\geq\left(1+\frac{\gamma_l\sigma_{N_l}^2}{2c_{ll}p_l}\right)\left(1+\sum_{i\neq l}\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right) \quad (46)$$

$$=1+\frac{\gamma_l\sigma_{N_l}^2}{2c_{ll}p_l}+\sum_{i\neq l}\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}+\frac{\gamma_l\sigma_{N_l}^2}{2c_{ll}p_l}\sum_{i\neq l}\gamma_l\frac{c_{li}p_i}{c_{ll}p_l} \quad (47)$$

$$\geq 1+\frac{\gamma_l\sigma_{N_l}^2}{2c_{ll}p_l}+\sum_{i\neq l}\gamma_l\frac{c_{li}p_i}{c_{ll}p_l} \quad (48)$$

$$=1+\gamma_l\frac{\sum_{i\neq l}c_{li}p_i+\frac{1}{2}\sigma_{N_l}^2}{c_{ll}p_l} \quad (49)$$

In Eqn. (46), the inequality comes from expanding the product on the left hand side of the equation and only keeping the two terms shown (the terms excluded after expanding are positive, resulting in the inequality). The inequality in Eqn. (48) once again comes from excluding a positive term in the sum. This gives the lower bound.

For the upper bound, once again consider the bound $1+x\leq e^x$. Applying this to the product terms gives

$$\begin{aligned} g_l(\gamma_l,\mathbf{p},\mathbf{u}_l,\mathbf{V}) &= e^{\frac{\gamma_l}{2}\frac{\sigma_{N_l}^2}{c_{ll}p_l}}\prod_{i\neq l}\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right) \\ &\leq e^{\frac{\gamma_l}{2}\frac{\sigma_{N_l}^2}{c_{ll}p_l}}\prod_{i\neq l}e^{\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}} \\ &= \exp\left\{\gamma_l\frac{\sum_{i\neq l}c_{li}p_i+\frac{1}{2}\sigma_{N_l}^2}{c_{ll}p_l}\right\} \end{aligned}$$

This concludes the proof.

□

Appendix II
Derivation of Standard Interference Function Properties

Standard interference functions were first defined in [28]. In this work, the authors showed that this class of functions can be used to do optimal power control, and it has nice convergence properties. The definition is given as follows:

Definition 1: An function $\mathbf{F}(\mathbf{p})$ is a standard interference function if the following properties are satisfied:

1) Positivity: $\mathbf{F}(\mathbf{p})>0$
2) Monotonicity: If $\mathbf{p}\geq\mathbf{p}'$, then $\mathbf{F}(\mathbf{p})\geq\mathbf{F}(\mathbf{p}')$
3) Scalability: For all $\kappa>1$, $\kappa\mathbf{F}(\mathbf{p})>\mathbf{F}(\kappa\mathbf{p})$

Theorem 2: Define

$$I_l(\mathbf{p})=\frac{\gamma_l\sigma_{N_l}^2}{2c_{ll}\log\beta_l}+\frac{p_l}{\log\beta_l}\sum_{i\neq l}\log\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right) \quad (50)$$

Then, the function $\mathbf{I}(\mathbf{p})=[I_1(\mathbf{p}),\ldots,I_L(\mathbf{p})]$ is a standard interference function.

Proof: A similar function and proof is given in [22]. Start with the first property, positivity. This can be proved by showing all the terms in Eqn. (50) are nonnegative, and at least one of them is positive. γ_l and $\sigma_{N_l}^2$ are >0 by how they are defined. From the constraints, $p_l\geq 0$ (the power can never be negative). Also, the logarithm of any number greater than 1 is positive. $\beta_l=(1-\alpha_l)^{-1}$, and $0<\alpha_l<1$, so $\beta_l>1$. Lastly, c_{li} is positive since $c_{li}=(\mathbf{v}_i^*\otimes\mathbf{u}_l)^H\mathbf{\Sigma}_{li}(\mathbf{v}_i^*\otimes\mathbf{u}_l)\geq 0$ since $\mathbf{\Sigma}_{li}$ is a positive semidefinite matrix. It is also safe to assume that $\mathbf{u}_l$ and $\mathbf{v}_l$ will be selected such that $c_{ll}>0$, since if $c_{ll}=0$ the expression for outage probability becomes undefined. Therefore, all the values in the first term of the sum in (50) are strictly greater than 0 and the second term in the sum is ≥ 0, so therefore $\mathbf{I}(\mathbf{p})>0$.

Monotonicity can be proved by showing $\frac{p_l}{\log\beta_l}\sum_{i\neq l}\log\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right)$ is monotonic in $\mathbf{p}$ (since the other term in Eqn. (50) is a constant with respect to $\mathbf{p}$). First, focus on only one term in this sum, since a sum of monotonically increasing functions will also be monotonically increasing. Then, consider the function

$$h(p_l,p_i)=\frac{p_l}{\log\beta_l}\log\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right) \quad (51)$$

If p_l is held constant, then $h(p_l,p_i)$ is monotonically increasing in p_i, since $c\log(1+kx)$ is monotonically increasing for a fixed $c>0$ and $k>0$. If p_i is now held constant, $h(p_l,p_i)$ takes the form

$$x\log\left(1+\frac{k}{x}\right)=x(\log(x+k)-\log(x)) \quad (52)$$

The function $h(p_l,p_i)$ is also monotonically increasing in p_l for $k>0$ since Eqn. (52) always has a positive derivative for $x>0$. Therefore, $\mathbf{I}(\mathbf{p})$ is monotonically increasing.

To prove scalability, consider a constant $\kappa>1$. Then

$$
\begin{aligned}
I_l(\kappa\mathbf{p}) &= \frac{\gamma_l \sigma_{N_l}^2}{2c_{ll}\log\beta_l} + \frac{\kappa p_l}{\log\beta_l}\sum_{i\neq l}\log\left(1+\gamma_l\frac{c_{li}\kappa p_i}{c_{ll}\kappa p_l}\right)\\
&= \frac{\gamma_l \sigma_{N_l}^2}{2c_{ll}\log\beta_l} + \kappa\frac{p_l}{\log\beta_l}\sum_{i\neq l}\log\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right)\\
&< \kappa\frac{\gamma_l \sigma_{N_l}^2}{2c_{ll}\log\beta_l} + \kappa\frac{p_l}{\log\beta_l}\sum_{i\neq l}\log\left(1+\gamma_l\frac{c_{li}p_i}{c_{ll}p_l}\right)\\
&= \kappa I_l(\mathbf{p})
\end{aligned}
$$

This concludes the proof. ■

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BIOGRAPHIES

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He is the holder of the Ericsson endowed chair in Wireless Access Networks and is the Director of the Center for Wireless Communications. His research group has received several paper awards. His paper received the best paper award at the 2000 speech coding workshop and his students have received student paper awards at both the 2005 and 2006 International conference on Acoustics, Speech and Signal Processing conference as well as the best student paper award at NIPS 2006. A paper he co-authored with B. Song and R. Cruz received the 2008 Stephen O. Rice Prize Paper Award in the Field of Communications Systems and a paper he co-authored with S. Shivappa and M. Trivedi received the best paper award at AVSS 2008. He also received the graduate teaching award from the graduate students in the Electrical Engineering department at UCSD in 1998. He was elected to the fellow grade in 2000 for his contributions in high resolution spectral estimation. Dr. Rao has been a member of the Statistical Signal and Array Processing technical committee, the Signal Processing Theory and Methods technical committee, the Communications technical committee of the IEEE Signal Processing Society. He has also served on the editorial board of the EURASIP Signal Processing Journal.

James R. Zeidler is a research scientist/senior lecturer in the Department of Electrical Engineering at the University of California, San Diego. He is a faculty member of the UCSD Center for Wireless Communications and the University of California Institute for Telecommunications and Information Technology. He has more than 200 technical publications and fourteen patents for communication, signal processing, data compression techniques, and electronic devices. Dr. Zeidler was elected Fellow of the IEEE in 1994 for his technical contributions to adaptive signal processing and its applications. He was co-author of the best student paper at the 2006 IEEE Personal, Indoor, and Mobile Radio Conference, received the Frederick Ellersick best paper award from the IEEE Communications Society at the IEEE Military Communications Conference in 1995, the Navy Meritorious Civilian Service Award in 1991, and the Lauritsen-Bennett Award for Achievement in Science from the Space and Naval Warfare Systems Center in 2000. He was also an Associate Editor of the IEEE Transactions on Signal Processing.

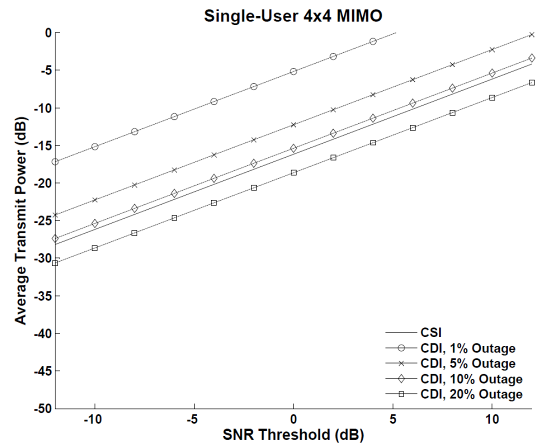

Fig. 1. Single User Case, SINR threshold vs Power

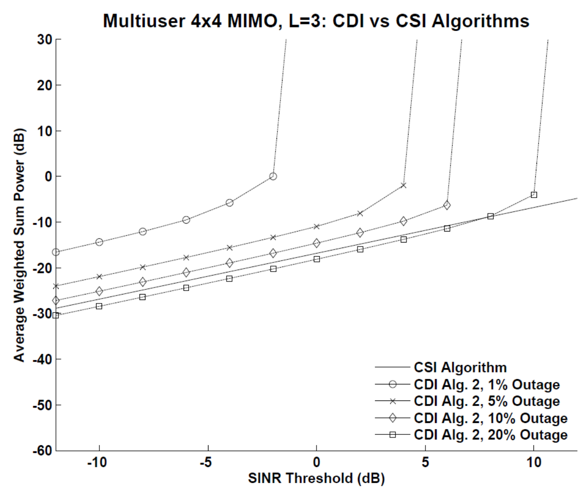

Fig. 2. Multiuser Case, SINR threshold vs Weighted Sum Power

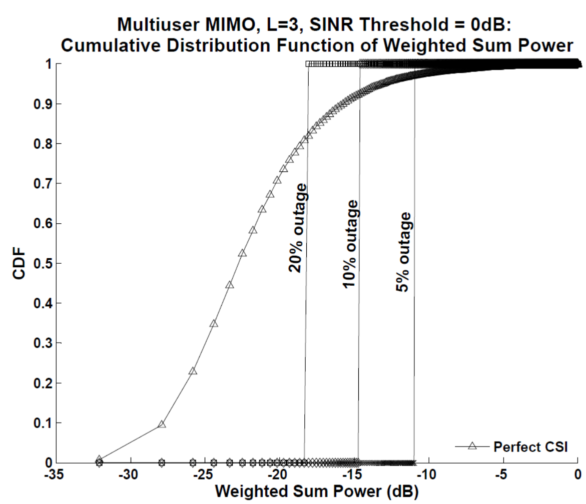

Fig. 3. Multiuser Case, CDF of Weighted Sum Power

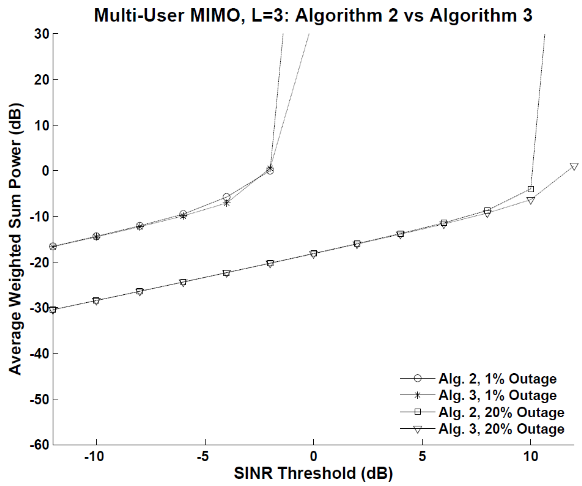

Fig. 4. Multiuser Case, Algorithm 2 vs Algorithm 3

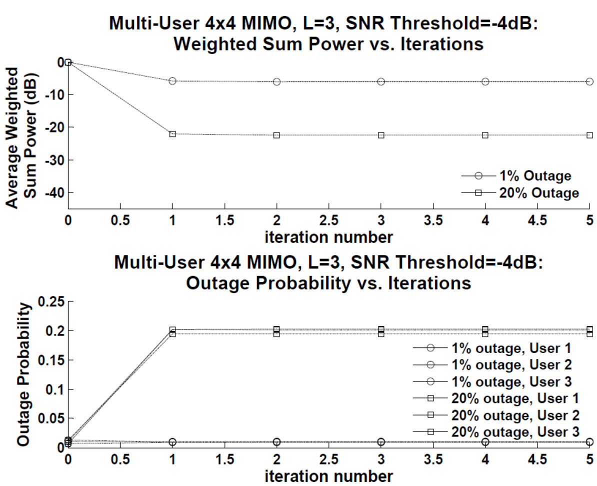

Fig. 5. Multiuser Case, Algorithm 2 Convergence